SCIENCE FOR GLASS PRODUCTION

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NUMERICAL MODELING AND ANALYSIS OF STRENGTH PROPERTIES IN GLASS

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This paper describes the methods for calculating strength properties of glass plates, such as deflection and stresses arising when static load is applied to glass. The specifics of various methods for calculating deflection and stresses are discussed. The problem is posed mathematically and the results of numerical solution of the corresponding boundary problem for a fourth-order partial differential equation (known as the biharmonic equation) are given. The numerical results are obtained with the use of the grid method developed by the authors.

Glass is one of the most common architectural materials widely used as transparent filling of light apertures. The progress in architecture, accumulated knowledge, and advanced technologies in the second half of the 20th century have led to a qualitative leap forward in glass application. New coatings and polymer materials for intermediate layers and new technologies for producing large glass sheets and bent and hardened glass contributed to glass being the dominant material in facades of contemporary building. Glass has gradually penetrated the area where its application had been impossible and even unthinkable: glass roofs, staircases, completely glazed facades without metallic carrier elements, and load-carrier structures.

Clearly the use of such "ambiguous" material as glass would have been limited without adequate scientific and engineering research and substantial investments. Making use of advantages of glass as a construction materials while decreasing the effect of its disadvantages is a complicated architectural and engineering problem, This is the reason for the fact that the most spectacular examples of original architecture with nontraditional and unusual application of glass have been designed in industrially developed countries. Furthermore, use of glass creates a certain aesthetic aspect that is popular in the West and not always accepted in our country. Obviously, some Russian specifics, such as our sharply continental climate, prevent direct copying of Western technical solutions. However, effective and efficient architectural solutions making use of glass are possible in Russia as well.

The purpose of this study is to model the behavior of glass structures, primarily, multiple glazing in buildings. This problem has to be solved consecutively, from simpler to more complex integrated cases. Since modeling is an integral stage of designing new buildings and of issuing expert evaluations for existing structures, a successful development of an applied program package could perceptibly influence the application of glass in domestic architecture. This would make it possible to simultaneously address the issues of safety, comfort, and other targets of contemporary architecture.

The european standard projects prEN 13474-1 and prEN 13474-2 are long known. They contain principles of technically simple analysis of stress and deflections arising in multiple and (or) single-compartment glazing. This makes its possible to compare glazing variants and to select glazing that can withstand loads expected in a particular building project.

The prEN 13474 method has the following advantages:

– simplicity: analysis can be performed even without a computer;

– many years of application;

– sufficient accuracy for comparing glazing variants;

– account of loads arising in a multiple glass unit under substantial variations of air temperature and pressure.

The authors have studied this method well and actively use it in practice [1 – 3]. When the specified standard was still being developed, researchers at the Institute of Glass developed a correlating method for analyzing mechanical characteristics of glazing variants.

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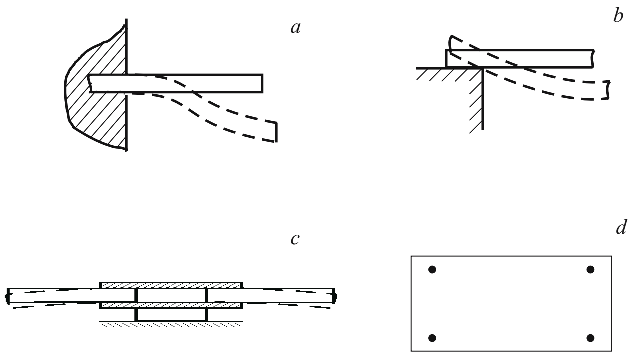


Fig. 1. Various types of glass fixing: *a*) restraint; *b*) support; *c*) point fastening; *d*) example of location of point fastening nodes.

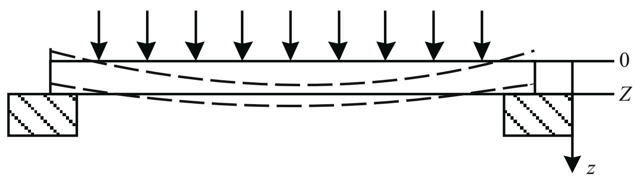


Fig. 2. Scheme of load application on a multiple window unit.

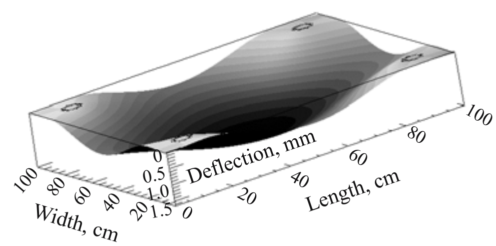


Fig. 3. Plate deflection in four-point fastening.

Thus, the problem appears to be already solved. However, this it not so, which is evidenced by the protracted delay in adopting this standard. The problem apparently consists in the fact that the prEN 13474 method has some essential drawbacks, which with respect to Russia can be divided into two categories; drawbacks that are common with the European countries, and the ones related to our specifics (in particular, Russian climate).

The main drawbacks of method prEN 13474 are as follows:

– the method does not consider such widely used method of glass fastening as point or spider-like fastening; this method can be conveniently used only for rectangular glass sheet fastened along their four, three, or two (opposite) sides;

– although the standard declares analysis for various types of loading, in reality it functions only for a uniformly distributed load, which does not satisfy contemporary requirements of design companies;

– detailed analysis is provided only for single-compartment double glazing, although the method could be refined and extended to a larger number of compartments;

– the only data obtained in analysis are the maximum deflection and the maximum and effective stress in glasses, but the distribution of these parameters is not provided;

– the method is in no way related to thermal and optical characteristics of glazing; up to a certain moment this is not significant; however, in severe conditions certain phenomena not predicted by the method have been repeatedly observed in practice.

Since we have encountered various glazing projects, we realized as early as the late 1990s the need to develop an original independent analysis method and software, which would be based on solving system of equations, as completely as possible, that describe glazing and that take into account all possible factors. Specialists in numerical modeling surely appreciate that this is too general a statement of the problem; however, we have been continuously specifying it and by now have achieved real results.

The development of calculation methods and programs has to begin with mathematical formulation of the problem, i.e., the problem has to be posed. It follows from the theory of strength of materials that in the general case the deflection of a plate of finite rigidity in the presence of chain forces is described by a complete system of von Karman differential equations. In a certain approximation, when the plate deflection is comparable to its thickness, this system is reduced to the biharmonic equation

$$\Delta^2 U = \frac{1}{D} q(x, y),$$

where U is the plate deflection; D is the cylindrical rigidity; $q(x, y)$ is the external load.

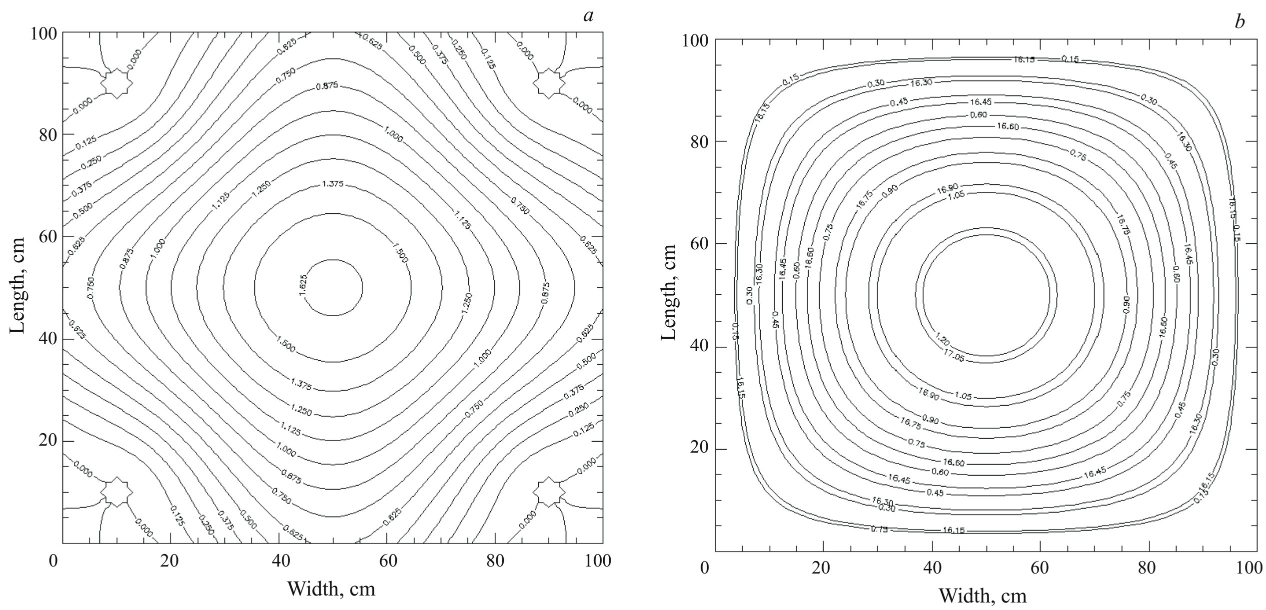
The equations are complemented by the respective boundary conditions depending on the solution range and on various methods of fixing a glass sheet along its perimeter. The most frequent fixation methods that have been analyzed are: restraint, bearing, also known as hinge fixing, point fastening (Fig. 1), and free (unfastened) edge.

The stresses in plates are described by a symmetric stress tensor whose components are expressed by the second and third spatial derivatives of the plate deflection value (particular formulas are not given because they are cumbersome).

It is clear that an exact solution of a "real" problem in the above case is impossible, since boundary or contour conditions cannot be expressed in an analytical form. Therefore, as usually happens in solving practical problems, approximation methods were used.

Approximation methods for solving problems can be split into two main groups:

– variational methods giving approximate analytical expressions of the desired function (transport function or inner force function);



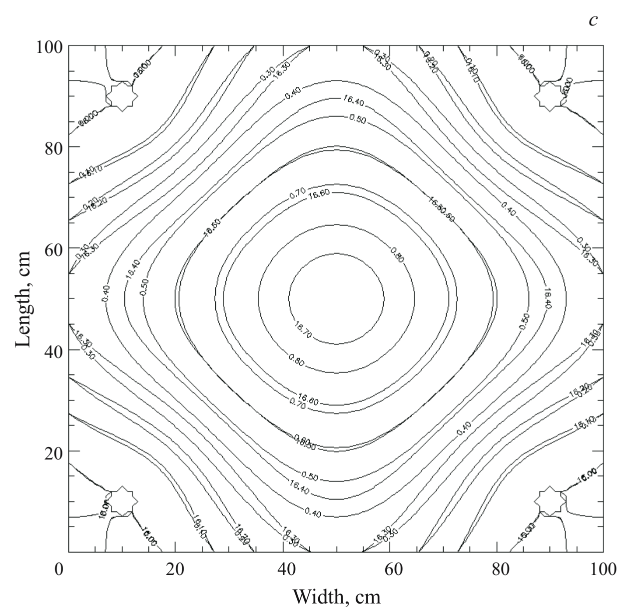


Fig. 4. Deflection of a plate in four-point fixing (*a*), deflection of a single-compartment window unit in support (*b*), and in four-point fixing (*c*). Level isolines.

– numerical methods yielding the values of the desired function for particular values of the argument.

The first group includes Ritz, Bubnov – Galerkin variational methods, Treffz method, etc. and the second group includes the grid method and its more perfect modification, i.e., the finite element method, as well as a number of graphic and semigraphic methods, for instance, the straight line method, the collocation method, etc.

The advantage of variational methods is that the problem is usually reduced to solving a system of two, three, rarely four equations, which gives a good approximation to the actual state of the structure. The disadvantage is that variational methods are limited by complicated contours and complex laws of external load distribution, since variational methods require analytical expressions, although in an approximate form, of external load, deformed elastic surface of the element, and other problem conditions.

Numerical methods are more universal than variational methods, since they do not need analytical expressions of problem conditions. However, numerical methods have a number of defects. In particular, to obtain satisfactory precision of solution, one may need to apply a rather fine grid to the area considered or split it into a large number of elements, which leads to increased dimensionality of the matrices of the systems of linear algebraic equations, which can be solved only on powerful computers. Furthermore, numerical methods on coarse grids can produce inaccurate solutions at the sites of application of concentrated forces, in the presence of acute angles, reinforcement, etc., where the smoothness of fields of variables is disturbed.

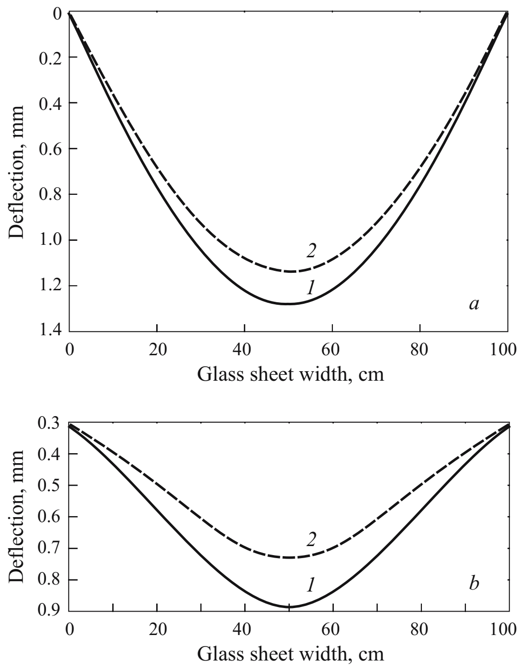


Fig. 5. Deflection in the middle of each plate of a single-compartment glass unit in support (*a*) and four-point fastening (*b*): *1* and *2*) deformation of upper and lower glass plates, respectively.

Thus, the choice of a method of analysis in considering a particular elastic system depends on the problem statement and initial conditions, as well as on available computer equipment. In our case for glazing modeling we adopted the grid method.

It should be noted that solving the considered problem requires a large memory and high processor capacity. Furthermore, some variants of the problem may contain a system with a poorly conditioned matrix. Only by optimizing memory and by regularization (whose description is outside the scope of the present paper) were we able to achieve a successful performance of the program complex on standard personal computers. Even so, a calculation may last for hours. It clearly depends on the workstation efficiency, but we are dealing with the most powerful among currently available workstations. However, advances in computer technologies give hope that in the near future this problem will cease to be acute.

By way of illustration, we present several plots (Figs. 2 – 5) obtained in calculations for glass plates and multiple glazing with various methods of fixing. It should be noted that these plots are constructed based on test calculations and the obtained values cannot be regarded as completely reliable. All results are obtained for plates of size 1000 × 1000 mm, glass thickness of 6 mm, distance between glass panes equal to 16 mm (in multiple glazing), and a uniformly distributed load equal to 800 N/m^2.

At present we are working on expanding the specified model and including so-called inner loads inside multiple glazing, which are directly related to thermal and optical parameters of multiple glazing in the climatic conditions of Russia.

In order to understand the meaning of the processes, note that heat in a multiple glass unit is transferred by different physical processes: conduction, radiation, and convection, The first mechanism can be easily suppressed, i.e., thermal conductivity can be decreased by increasing the glass thickness and the distance between the panes; the second factor is alleviated using various energy-saving coatings. The remaining factor, i.e., convection, is the most difficult process for modeling; moreover, the dependence of the intensity of heat transfer by convection on the distance between the glass panes is inverse to the heat conductivity: as the distance between the glass sheets grows, the contribution of convective flows to the thermal fields in the glazing unit perceptibly grows.

As the temperature inside the glazing compartment decreases, the pressure inside the compartment decreases as well, which leads to the deformation of glasses and their deflection toward each other. This mechanical effect, in turn, may significantly modify the thermal situation in the glazing unit compartment: the scheme of convective gas flows changes. In practice, under considerably cooling, the central part of the glass unit may be frozen. In order to reflect such effects in the model, it should be complemented by analysis of gas flows inside the glazing unit compartment. There are reasons to believe that the correct approach implies joint modeling of thermal and strength properties of glazing, since in the Russian conditions they perceptibly influence each other.

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